Vibration Confinement in a Generic Launch Vehicle

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Introduction

CONTROL of vibration and maintainance of position alignment in sensitive parts of a launch vehicle are critical to the mission. For example, mounting systems for rate gyro packages demand precision alignment to the axis of the vehicle and dynamic characteristics that limit the sinusoidal and random responses during test conditions within specified limits. In a recent acoustic test of a mounting system, along with a part of the rocket structure, it was observed that the root mean square values of the vibration responses were beyond the specification in some frequency bands. This led to the primary motivation for this work: vibration control and alignment maintenance at critical locations in a launch vehicle. A generic launch vehicle (GLV) is chosen, whose equipment bay (EB) houses the main inertial platform. Figure 1 shows the location of the EB on the GLV. The objective is to reduce the disturbance in the attitude at the EB due to elastic deformation by active control. The vibration energy is redistributed by the technique of vibration confinement,[1] which enables the response at the EB to reach its steady state faster than in the remaining portion of the structure.

Mathematical Modeling

The GLV is assumed to have two axes of symmetry in the pitch and the yaw planes. A two-degree-of-freedom per node beam element (one translation and one rotation) is used for finite element discretization, and 38 elements and 78 degrees of freedom are chosen. The element stiffness and mass matrices are evaluated using Hermite cubic interpolation functions. They are then used to form the global stiffness and mass matrices by appropriate connectivity information. The mass and stiffness matrices are (78×78). M has full rank, whereas K has a rank of 76, indicating its semidefiniteness, which stems from the presence of the two rigid-body modes. The eigenvalue problem associated with the K and M matrices is solved to obtain the natural frequencies and mode shapes. The first 12 frequencies are $(0\ 0\ 1.955\ 6.197\ 9.2\ 15.17\ 24.54\ 27.75\ 53.02\ 55.02\ 69.75\ 74.76)^T$ Hz. The size of the system is reduced by Guyan's[2] reduction. The order of the reduced-order model is resolved by the Kimura condition[3] according to which the number of actuators and sensors dictate the size of the reduced-order model. In the GLV, four engine actuators are assumed to be located at nodes 6, 11, 19, and 28. A minimum of two additional actuators are required to generate a torque for shape control of the EB. To select the best reduced-order model, three reduced-order models are constructed: (12×12), (6×6), and (5×5). Both the (12×12) and the (6×6) models are found to adequately represent the original system characteristics. Implementation of a controller in the (12×12) system requires positioning of 12 actuators on the GLV, some of whose locations are physically infeasible for actuator mounting, such as on a motor case. Therefore, a (6×6) reduced-order model is selected for controller design.

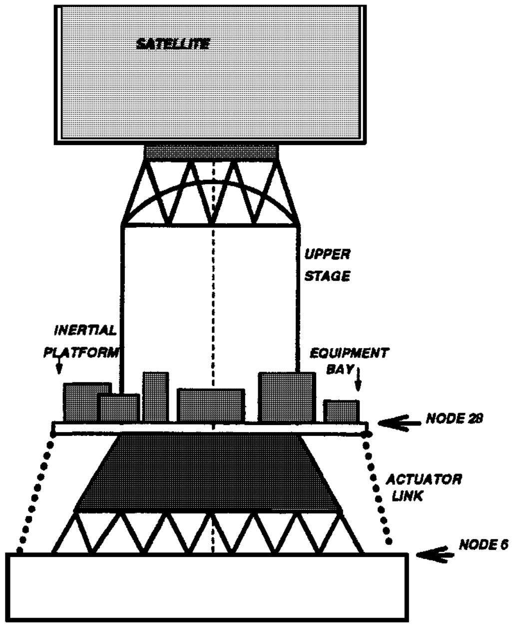

Fig. 1 Schematic of the EB on the GLV.

The mass and stiffness matrices of the (6×6) reduced-order model are

$$M \times 10^{-5} = \begin{pmatrix} 0.6851 & -0.0528 & 0.1334 & -0.0618 & -0.7853 & 0.3439 \\ & 0.4441 & 0.1760 & -0.0803 & 0.0818 & -0.0358 \\ & & 1.3518 & 0.0097 & -0.1911 & 0.0837 \\ & & & 0.4926 & 0.0885 & -0.0388 \\ & \text{symmetric} & & & & 0.2469 \end{pmatrix}$$

$$K \times 10^{-10} = \begin{pmatrix} 0.8361 & 0.0051 & -0.0019 & 0.009 & -0.8402 & 0.9834 \\ & 0.0025 & -0.003 & 0.0016 & -0.0063 & 0.0028 \\ & & 0.0052 & -0.0031 & 0.0028 & -0.0012 \\ & & & 0.0018 & -0.0013 & 0.0006 \\ & \text{symmetric} & & & & 1.1764 \end{pmatrix}$$

For the reduced-order model, actuators and sensors are assumed to be located at degrees of freedom 11, 21, 29, 37, 55, and 56. Degrees of freedom 55 and 56 correspond to control of the displacement and the slope at the EB, respectively.

Controller Design and Simulation

The equations of motion of the system are given by $M\ddot{x} + C\dot{x} + Kx = F$, where the control force is $F = -\bar{M}\ddot{x} - \bar{C}\dot{x} - \bar{K}x$. Incorporate the control force into the equation of motion and transform the coordinates through the equation $x = Q\eta$ to decouple the equations. The control strategy to convert the original vibratory modes into a set of modes that allow the vibration energy to be trapped in a prescribed region of the structure is described in detail by Choura.[1] The rows of the Q matrix are chosen depending on the way the vibration energy is required to be relocated in the structure. Small elements are allocated to those rows corresponding to coordinates in which the desired energy levels are low. Q is assumed to take the form

$$Q = \begin{pmatrix} 1 & 1 & \cdots & 1 \\ s_{21}\gamma_1 & \gamma_1 & \cdots & \gamma_1 \\ s_{31}\gamma_2 & s_{32}\gamma_2 & \cdots & \gamma_2 \\ \cdots & \cdots & \cdots & \cdots \\ s_{(n-1)1}\gamma_{n-2} & s_{(n-1)2}\gamma_{n-2} & \cdots & \gamma_{n-2} \\ s_{n1}\gamma_{n-1} & s_{n2}\gamma_{n-1} & \cdots & \gamma_{n-1} \end{pmatrix} \tag{17a}$$

Presented as Paper 98-4930 at the AIAA/USAF/NASA/ISSMO 7th Symposium on Multidisciplinary Analysis and Optimization, St. Louis, MO, Sept. 2–4, 1998; received Oct. 29, 1998; revision received Jan. 22, 1999; accepted for publication Jan. 22, 1999.

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where the scalar γ_j sets the vibration energy of the $(j+1)$th mass with respect to the first mass. If the $(j+1)$th mass is considered to be a sensitive part, then γ_j should be less than one. On the other hand, if the first mass is considered to be a sensitive part, then γ_j should be more than one. The s_{ij} that multiply the lower diagonal elements of Q are introduced to preserve the pattern of the sign changes that exist in second-order mechanical systems. They are defined by $s_{ij} = -1$ if $(i+j)$ is odd and $s_{ij} = +1$ if $(i+j)$ is even.

The gains are given by[1] $\bar{M} = 0$, $\bar{C} = MQ\Gamma Q^{-1} - C$, and $\bar{K} = MQ\Lambda Q^{-1} - K$, where Λ is the set of targeted closed-loop eigenvalues and Γ is the diagonal matrix that specifies the closed-loop damping values. The physical damping matrix C was not computed in this study because the structural damping is very low when compared to $\bar{C}$, the damping introduced by the velocity feedback into the system.

Full-Order Controller

The controller design for the full-order plant is carried out with all states fed back. It is assumed that all state measurements are available as sensor outputs. This part will serve as the base for the controllers designed with the reduced-order models for comparison.

The size of the mass and the stiffness matrices is (78×78). The size of the Q matrix is also (78×78). This matrix is formulated from Eq. (1) as follows:

$$Q = \left(\begin{array}{c|cccccc} & \text{1st} & \text{2nd} & & & \text{77th} & \text{78th} \\ \hline \text{1st} & +1 & +1 & \cdots & \cdots & +1 & +1 \\ \text{2nd} & -1 & +1 & \cdots & \cdots & +1 & +1 \\ \text{3rd} & +1 & -1 & \cdots & \cdots & +1 & +1 \\ & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \text{55th} & +0.1 & -0.1 & \cdots & \cdots & +0.1 & +0.1 \\ \text{56th} & -0.1 & +0.1 & \cdots & \cdots & +0.1 & +0.1 \\ & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \text{78th} & -1 & +1 & \cdots & \cdots & +1 & +1 \end{array}\right)$$

Locations 55 and 56 correspond to the displacement and slope at the EB, where the magnitude of the eigenvector is reduced to $\frac{1}{10}$th.

The first two modes, being rigid-body modes, have zero frequency. For the closed-loop system, the first two eigenvalues are selected to be larger than zero. Eigenvalues for the higher modes are complex. The damping ratio ς for the closed-loop system is chosen to be 0.205 for the first mode and 0.58 for the last mode. For the other modes, the value is incremented from the lower to the higher modes.

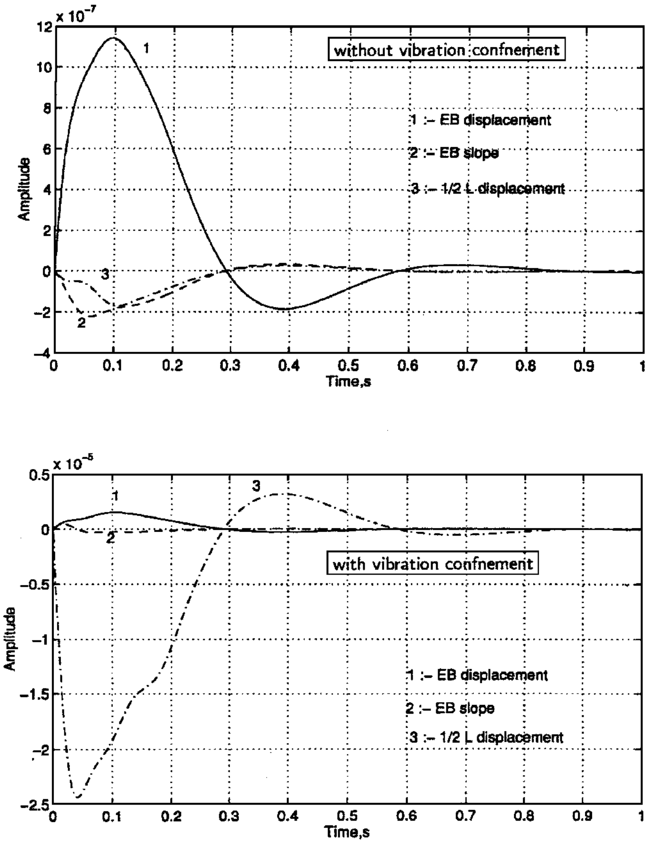

Fig. 2 Impulse response of the (6 × 6) reduced-order model.

The closed-loop system with the controller is simulated with the outputs being the displacement and the slope at the midpoint of the GLV and at the EB. It is seen that in the absence of vibration confinement, all of the four states approach the steady state at the same time. With vibration confinement, steady state is reached at the EB location $1\frac{1}{2}$ times faster than at the midpoint of the GLV.

Reduced-Order Controller

For the (6×6) reduced-order model, the target eigenvector matrix Q is formulated in the same way as in the case of full-order controller.

The displacement and velocity feedback gains for the reduced-order system are

$$\bar{K} \times 10^{-10} = \begin{pmatrix} -1.9100 & 2.7000 & 3.3100 & -0.0117 & 238.0000 & -116.0000 \\ -0.0018 & 1.47000 & 1.8100 & 0.0020 & 87.6000 & -11.5000 \\ -1.1600 & 3.9300 & 4.8600 & 0.0250 & 304.0000 & -115.0000 \\ 0.1030 & 0.2270 & 0.3060 & 0.0171 & 0.0930 & 13.9000 \\ 2.0900 & -3.0800 & -3.7700 & 0.0137 & -273.0000 & 135.0000 \\ -1.5200 & 1.3500 & 1.6500 & -0.0060 & 119.0000 & -59.5000 \end{pmatrix}$$

$$\bar{C} \times 10^{-7} = \begin{pmatrix} -0.2210 & 1.2000 & 1.5000 & -0.1340 & 88.7000 & -58.4000 \\ -0.0391 & 0.7430 & 0.7630 & 0.0098 & 33.8000 & -0.5310 \\ -0.5340 & 1.6600 & 2.4700 & 0.0300 & 135.0000 & -58.8000 \\ 0.0741 & 0.0775 & 0.1600 & 0.1640 & -7.3200 & 11.7000 \\ 0.2720 & -1.3600 & -1.7200 & 0.1580 & -103.0000 & 68.8000 \\ -0.1150 & 0.5990 & 0.7580 & -0.0696 & 44.8000 & -29.6000 \end{pmatrix}$$

They are seen to be very high (of the order of 10^{10} for displacement feedback and 10^7 for velocity feedback), which implies that large forces are required to achieve the required vibration control. This